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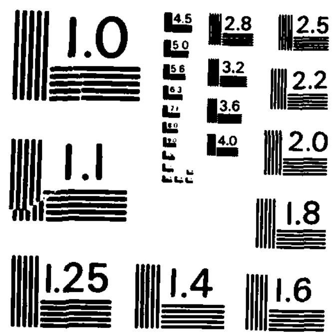

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R. A. LOCKHART, F. O'REILLY and M. A. STEPHENS

TECHNICAL REPORT NO. 369
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Tests for the Extreme Value and Weibull Distributions Based on Normalized Spacings

By

R.A. Lockhart, F. O'Reilly and M.A. Stephens

1. INTRODUCTION.

In this article we compare tests of the null hypothesis H_0 that a sample, which may be censored at one or both ends, comes from an extreme-value distribution with unknown location and scale parameters; such tests can also be used to test that observations come from a Weibull distribution with origin zero, and scale and shape parameters unknown. Such tests of H_0 are important in many applications, in particular in reliability theory and survival analysis.

We first discuss a class of tests based on normalized spacings, that is, on the spacings between the ordered observations, standardized by dividing by known constants. Such spacings are transformed to a set of values z_i between 0 and 1, and test statistics are calculated from these. Two such tests have been suggested by Mann, Scheuer and Fertig (1973) and Tiku and Singh (1981); these are the tests based respectively on the median and on the mean $\bar{Z}$ of the z_i. Asymptotic distributions for these statistics are given; it is also shown that they may be not consistent or biased. We propose the use of the Anderson-Darling statistic A^2 based on the z_i, and asymptotic distribution theory and percentage points are given for various censoring patterns.

A^2 gives a consistent and unbiased test. Finally we compare the above three statistics for power with two others which have been proposed for testing the Weibull or extreme-value distributions.

The practical application of the normalized spacings tests is discussed in the next section; distribution theory follows in Section 3, and the power comparisons, based on Monte Carlo studies, are described in Section 4. The recommended test overall is A^2; although for the alternatives considered $\bar{Z}$ has good power, it is less attractive because of its possible non-consistency. Another version of A^2 introduced by Stephens (1977) does well, but is not available for censored samples.

2. TEST PROCEDURES.

2.1. Calculations.

Let $x_{(k)}, x_{(k+1)}, \ldots, x_{(k+t+1)}$ be $t+2$ order statistics of a random sample of size n which is left and right Type 2-censored. We wish to test the null hypothesis

H_0: the original sample comes from the extreme-va[illegible] istribution

$$F(x) = 1 - \exp[-\exp\{(x-\alpha)/\beta\}], \qquad -\infty < x < \infty \quad . \tag{1}$$

The test statistics are calculated as follows.

(a) Calculate $t+1$ normalized spacings

$$y_j = \{x_{(k+j)} - x_{(k+j-1)}\}/(m_{k+j} - m_{k+j-1}), \qquad j = 1, \ldots, t+1 \tag{2}$$

where m_i is the expected value of the i-th order statistic of a random sample from $F(x)$ in (1) above, with $\alpha = 0$ and $\beta = 1$. Values of m_i have been tabulated by White (1967) and by Mann (1968). Values of the difference $m_i - m_{i-1}$ are tabulated for $n = 3(1)25$, by Mann, Scheuer and Fertig (1973). For $n > 25$ an approximation for m_i has been given by Blom (1958, p. 73 ff.):

$$m_i = \log[-\log\{1-(i-0.5)/(n+0.25)\}] \quad ,$$

where log refers to natural logarithm. This approximation appears to be adequate for the present use.

(b) Let T_i be the partial sum $T_i = \Sigma_{j=1}^{i} y_j$ (so that T_{i+1} is the sum of all the y_j) and define

(3) $$z_i = T_i/T_{i+1}, \quad i = 1,\ldots,t .$$

Note that the z_i are in ascending order.

(c) The three statistics here considered are A^2, S and $\overline{Z}$.

(4) $$A^2 = -t - t^{-1}\left[\sum_{i=1}^{t} (2i-1)\{\log z_i + \log(1-z_{t+1-i})\}\right];$$

A^2 is the Anderson-Darling statistic calculated from the z-values. Also

(5) $$S = 1 - z_s ,$$

where $s = (t+2)/2$ if t is even, and $s = (t+1)/2$ if t is odd; and

(6) $$\overline{Z} = \sum_{i=1}^{t} z_i/t .$$

Clearly $\overline{Z}$ is the mean of the z_i and z_s is essentially the median, so that S is equivalent to the median.

2.2. Application of the tests.

For the test based on A^2, H_0 will be rejected if A^2 exceeds the value given, for the desired level α, in Table 1. The table gives asymptotic critical points for A^2, but can be used with high accuracy for $n \geq 20$. The table is entered at $p = k/n$ and $q = (k+t+1)/n$; the table is easily interpolated for values of p and q not given.

Statistic S was introduced by Mann, Scheuer and Fertig (1973; see also Mann, Fertig and Scheuer, 1971) for right-censored samples only: note that their $m = t+2$. A statistic proposed by Tiku and Singh (1981), and there called Z_W^*, can be shown to be $2\bar{Z}$.

In general, to guard against all alternatives, both S and $\bar{Z}$ should be used as two-tail tests. However, Mann, Scheuer and Fertig were testing for the Weibull distribution (see below) and intended S to be used as a one-tail test, with large values significant, for the alternatives they wished to detect; they give Monte Carlo points only for the upper tail of S, for $3 \leq n \leq 25$. The large-sample distribution of S is discussed in Section 3 below.

Tiku and Singh suggested an accurate normal approximation for Z_W^* which would give for $\bar{Z}$ the normal approximation $\bar{Z} \approx N(0.5,V)$; V depends on the m_i, and also on the variances and covariances of standard extreme-value order statistics. The calculation of V is quite complicated, and a simpler normal approximation is given in Section 3.

2.3. Application to testing for the 2-parameter Weibull distribution.

The above tests may be used for the 2-parameter Weibull distribution with unknown scale parameter δ and shape parameter γ

$$(7) \qquad F(w) = 1 - \exp\{-(w/\delta)^{\gamma}\}, \quad w > 0 ;$$

the random variable $x = \log w$ has the distribution (1) with $\alpha = \log \delta$ and $\beta = 1/\gamma$. Thus a censored sample $w_{(k)},\ldots,w_{(k+t+1)}$, to be tested to have distribution (7), is transformed to $x_{(k)} = \log w_{(k)}$, $x_{(k+1)} = \log w_{(k+1)},\ldots,$ and the x-set is tested to come from distribution (1).

2.4. Example.

Table 2 gives values of a right-censored sample of ignition times, given by Mann, Scheuer and Fertig. The sample size is $n = 22$, and times $w_{(1)},\ldots,w_{(15)}$ are available, to test that the distribution of w is two-parameter Weibull. The table also gives values of $\log w$, the normalized spacings y, and the values z. The value of A^2 is 1.878, and reference to Table 1, with $p = 0$ and $q = 15/22$ suggests a significance level of about 10%. The median is 0.3394 and $S = 0.661$; reference to the Mann, Scheuer and Fertig tables show S to be significant at about the 11% level. Statistic $\bar{Z}$ equals 0.380; this is significant at about the 12% level in the lower tail. If, as is suggested by Mann, Scheuer and Fertig, S should be used as a one-tail statistic, then $\bar{Z}$ would be used similarly, with low values of $\bar{Z}$ (as here) leading to rejection of the hypothesis that the ignition times have a two-parameter Weibull distribution, in favor of a distribution like a three-parameter Weibull alternative with positive origin. For a two-tail omnibus test, the significance levels of S and $\bar{Z}$ will be doubled, and A^2 is then more sensitive than either of these. In general, the power studies in Section 4 shows A^2 and $\bar{Z}$ to be more sensitive than S.

3. DISTRIBUTION THEORY AND TABLES.

3.1. Normalized spacings: asymptotic theory.

If the original sample $x_{(k)},\ldots,x_{(k+t+1)}$ were from an exponential distribution with origin 0, and mean β, written $\mathrm{Exp}(0,\beta)$, the normalized spacings would be i.i.d. exponential $\mathrm{Exp}(0,\beta)$ and the z_i would have exactly the joint distribution of uniform order statistics from a sample of size t; an individual z_i would have a Beta distribution. More generally, and subject to important conditions particularly affecting the extreme spacings, suitably separated normalized spacings from any continuous distribution are asymptotically independent and exponentially distributed with mean 1; (see Pyke, 1965, for rigorous and detailed results). The conditions on this result are sufficiently strong, however, that the transformed values z_i, when the $x_{(j)}$ come from a distribution other than the exponential, must not be assumed to be distributed as uniform order statistics, even asymptotically, for the purpose of finding distributions of test statistics. The situation is somewhat similar to tests involving unknown parameters when EDF statistics are used; even if the parameters are estimated efficiently, the asymptotic distributions of test statistics are not the same as they are when the parameters are known.

The authors have examined the asymptotic properties of normalized spacings elsewhere (Lockhart, O'Reilly and Stephens, 1984), and from these results the asymptotic distributions of A^2, S and $\bar{Z}$ have been derived for a number of parent populations, in particular the extreme-value, normal and logistic populations. Mathematical details of the distribution theory have been omitted from the present article, where the emphasis is on practical aspects of the tests.

Statistic A^2. The asymptotic null distribution of A^2 is a sum of weighted χ^2 variables, with weights depending on the parent population and also on the censoring levels. The points given in Table 1 were found by calculating the weights for the extreme-value distribution; the percentage points can then be approximated very accurately, as described by Lockhart, O'Reilly and Stephens (1984). Monte Carlo results for finite n suggest that the points in Table 1 can be used to good accurancy for $n \geq 20$. It is worth noting that, if the z_i were uniform order statistics, the points in Table 1 for $p = 0$ and $q = 1$, would be those for A^2, Case 0, given for complete samples in Stephens (1974), whereas in fact they are quite different; in the next section we see that there are differences also for S and $\bar{Z}$. These differences demonstrate the remark above, that the z_i cannot be treated as uniform order statistics, even asymptotically.

Statistics S and $\bar{Z}$. Under regularity conditions, the asymptotic distributions of both S and $\bar{Z}$ under the null hypothesis are normal with mean 0 and variance determined by the parent population and the censoring levels. For S, this leads to the following large sample approximation. Let

$$(8) \qquad S^* = C\, t^{\frac{1}{2}}(S-0.5) \quad \text{if } t \text{ is odd;}$$

$$= C\, t^{\frac{1}{2}}[S-0.5+1/\{2(t+1)\}] \quad \text{if } t \text{ is even ;}$$

percentage points of S^* are then well-approximated by those of the standard normal distribution for $t \geq 20$. The values of C can be found from the formula for the asymptotic variance of S, given by Lockhart, O'Reilly and Stephens (1984). For the extreme-value distribution, and for a complete sample, $C = 2.233$; for a right-censored sample the value of C diminishes steadily towards 2.0, as q, the proportion of sample uncensored, diminishes from 1 to 0.

If the z_i were uniform order statistics the distribution of S in (5) would be a Beta distribution with parameters $(t+1)/2$, $(t+1)/2$ when t is odd, and $t/2$, $(t+2)/2$ when t is even. Mann, Scheuer and Fertig (1973, p. 390; also see Mann and Fertig, 1975, p. 239) suggest the use of this approximation for large n, and find good agreement with Monte Carlo points for S even for quite small n. However, for large samples, the Beta approximation gives the incorrect result that $S_1 \equiv 2t^{\frac{1}{2}}(S-0.5)$ is approximately standard normal. This result would be true regardless of the censoring pattern, and we shall see a similar result for $\bar{Z}$ below. Thus the Beta approximation essentially gives $C = 2$ in (8) above, instead of $C > 2$; this leads to a conservative test, with an error in significance level which depends on the censoring pattern. For example, for a right-censored sample, the error grows larger with the fraction of available observations, and with n.

For statistic $\bar{Z}$, the normal approximation is that, for large t,

$$Z^* = Kt^{\frac{1}{2}}(\bar{Z}-0.5) \tag{9}$$

has a standard normal distribution, where K depends again on the parent population and on the censoring. Table 3 gives values of K for some censoring fractions p and q. (If the z_i were uniform order statistics, K would always be $\sqrt{12} = 3.464$.) Table 3 permits some limited comparisons with critical values for $2\bar{Z}$ given by Tiku and Singh, p. 911. For example, for $n = 20$, with $k=3$ and $k+t+1=18$, we take $p = .15$ and $q = .85$, and interpolation in Table 3 suggests $K \approx 3.77$; using $t = 14$ in (9), we have for the upper 2.5% point for $\bar{Z}$ the value 0.639; Tiku and Singh give .641, and the difference in α-levels is 0.002. Similar comparisons indicate that (9) may be used with good accuracy for $n \geq 20$.

In Lockhart, O'Reilly and Stephens (1984) some examination is made also of the distributions of S and $\bar{Z}$ when the null hypothesis is false; these are again asymptotically normal, and for tests of normality against some symmetric alternatives asymptotic power can be calculated. The calculations show that $\bar{Z}$ and S may sometimes not be consistent; used with one tail, S can also be biased. We return to these possibilities in the next section.

4. PERFORMANCE OF TEST STATISTICS.

4.1. Power Comparisons.

A large Monte Carlo study has been conducted to compare the three statistics above; also included were the correlation coefficient R, between the values $x_{(i)}$ and $H_i = \log[-\log\{1-i/(n+1)\}]$, and the Anderson-Darling statistic A^2, used as described in Stephens (1977). The R test is based on the fact that $x_{(i)}$ plotted against m_i should be close to a straight line; H_i is a convenient approximation to m_i, and a low value of R indicates a bad fit. Tables for R have been given by Smith and Bain (1976), and more extensive tables of $n(1-R^2)$, by Stephens (D'Agostino and Stephens, 1985). For the Stephens (1977) A^2 test, the parameters α and β in (1) must be estimated from the sample by maximum likelihood; then the probability integral transform $\tilde{z}_i = F(x_{(i)})$, $i = 1,\ldots,n$, is made, using these estimates in $F(\cdot)$. Finally A^2 is calculated from the $\tilde{z}_i$ using the formula (4) with $t = n$ and $\tilde{z}_i$ replacing z_i. This is the customary way to deal with unknown parameters when using EDF statistics, and special tables of critical points must be produced. These are given in Stephens (1977), for complete samples only, for A^2 and two other EDF statistics, W^2 and U^2; points for the Kolmogorov D and Kuiper V are given in Chandra, Singpurwalla and Stephens (1981). Experience with tests for other distribtuions suggests that this version of A^2 is likely to be a good statistic in terms of power. Mann, Scheuer and Fertig also used EDF statistics in this way in their power comparisons using S. In the present study, only complete samples were compared; our alternative distributions included those used by Mann, Scheuer and Fertig, and also others examined by Littell, McClave and Offen (1979) and by Tiku and Singh (1981).

Table 4 reports the power results for complete samples of size $n = 20$ and $n = 40$ using 5% tests, and using 5000 Monte Carlo samples for each run. A^2 refers to the A^2 suggested in this article, and A_1^2 to A^2 using estimated parameters as described above.

Mann, Scheuer and Fertig had in mind a test of the two parameter Weibull distribution, which in the form (7) has a long tail to the right, with the alternative a three parameter Weibull distribution

$$F(w) = 1 - \exp[-\{(w-\alpha)/\delta\}^{\gamma}], \qquad w > \alpha \; ; \tag{10}$$

this is referenced in Table 4 as $W(\alpha,\delta,\gamma)$; in particular α was assumed to be positive. For this test, S will be large when the alternative is true and so a one-tail test was suggested with S. It appears that the one-tail test is appropriate for most of the alternatives here listed. However, as was pointed out by Tiku and Singh, such a test will lead to bias against some alternatives, so we record results both for S used with one tail $(S(1))$ and with two tails $(S(2))$. Results are available also for other test levels; they give a similar picture. Studies were also made with sample size 10; these agree, for S and A_1^2, and to within sampling fluctuations, with power tables given by Mann, Fertig and Scheuer. The power results reported here also agree with those of Tiku and Singh (1981).

4.2. Comments on Table 4.

(a) Of the three statistics based on normalized spacings, A^2 and $\bar{Z}$ outperform S, even when S is used with a one-sided test. $\bar{Z}$ does well, confirming the results of Tiku and Singh (1981); it seems intuitively reasonable that $\bar{Z}$ makes better use of all the observations z_i than does S, for most patterns of z-values. A^2 and $\bar{Z}$ are very close in terms of power, although A^2 is superior for the heavy-tailed Cauchy alternative.

(b) A_1^2, is occasionally better than A^2 and $\bar{Z}$, although to offset this is the fact, pointed out by Mann, Scheuer and Fertig and again by Tiku and Singh, that A_1^2 is somewhat difficult to use because of the necessity to estimate parameters; this objection does not apply to A^2 calculated directly from the same z_i which must be derived to get S and $\bar{Z}$, and the formula for A^2 is simple and easily programmable even on small calculators.

(c) The correlation coefficient R^2 is occasionally powerful but sometimes very weak; this test might improve if H_i were replaced by the correct m_i (tables for R have been given by Gerlach (1979) for this case) but it seems unlikely that it will reach the power levels of A^2 and $\bar{Z}$ overall.

(d) Bias in S(1) is seen in the log χ_1^2 alternative to the extreme-value distribution (or the χ_1^2 alternative to the two-parameter Weibull). For the particular problem of testing two-parameter Weibull against three-parameter Weibull, with right-censored data, Mann and Fertig (1975) subsequently proposed a statistic which they show to be more powerful than S; this statistic is their $P_{k,m}$, and is equivalent to using z_s above, with s much closer to t than t/2. It seems likely that there would be some alternatives for which $P_{k,m}$ also would be biased and not consistent.

4.3. Bias and non-consistency: further remarks.

The results in Table 4 show that $\bar{Z}$ might be regarded as a serious competitor to A^2 for the alternatives given, but it has already been suggested, based on results in testing normality, that there will be some distributions against which tests based on $\bar{Z}$ (or S) might not be consistent; then even a very large sample will not detect such an alternative distribution, say F^*, for X, with high probability, as one would wish. We briefly illustrate this possibility, using $\bar{Z}$; similar remarks would apply to S. When the sample X comes from F^*, and $\bar{Z}$ is calculated as in Section 2.1, suppose the asymptotic mean μ of $\bar{Z}$ is 0.5 and let the asymptotic variance of $t^{\frac{1}{2}}(\bar{Z}-0.5)$ be $1/K_1^2$; then the normal approximation (9) holds to high accuracy, with K_1 replacing K. Suppose a 5% test is made; it is easily shown that the probability of rejecting H_0: the extreme-value hypothesis for X, is twice the area in the standard normal tail beyond the value $1.96K_1/K$, and this will be a constant not necessarily near 1 - it could even be less than 5%. A distribution for which this situation arises, where $\bar{Z}$ has the asymptotic mean $\mu = 0.5$, is $F^*(x) = 1-(1-x)^c$, with $c = 1/\sqrt{2}$; the calculations to show this are complicated and are omitted here. In statistical practice, it will not often occur that $\bar{Z}$ has exactly $\mu = 0.5$, but even if, for a given F^*, the value is close to

0.5, an enormously large sample would be needed to give good power using $\bar{Z}$ against this alternative. Such problems do not appear to have arisen for the alternatives used in Table 4, but in general non-consistent tests are undesirable, and the possibility of such occurrences must make $\bar{Z}$ less attractive than A^2 overall.

Acknowledgements.

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TABLE 1

Asymptotic percentage points for A^2. The table is entered at $p = k/n$ and $q = (k+t+1)/n$.

Left censoring point, p	Right censoring point, q	Significance level α						
		0.25	0.20	0.15	0.10	0.05	0.025	0.01
0	1	1.016	1.138	1.300	1.535	1.957	2.398	3.000
0.0	0.75	1.159	1.302	1.492	1.770	2.267	2.787	3.498
0.0	0.50	1.202	1.354	1.555	1.849	2.376	2.927	3.682
0.0	0.25	1.229	1.386	1.594	1.898	2.444	3.015	3.797
0.25	1.0	1.027	1.150	1.312	1.549	1.972	2.413	3.018
0.25	0.75	1.187	1.336	1.532	1.819	2.333	2.870	3.605
0.25	0.50	1.231	1.388	1.596	1.901	2.447	3.018	3.800
0.50	1.0	1.051	1.177	1.345	1.589	2.025	2.481	3.105
0.50	0.75	1.224	1.380	1.586	1.887	2.428	2.993	3.767
0.75	1.0	1.081	1.213	1.387	1.641	2.096	2.571	3.222

TABLE 2

(a) 15 smallest values of 22 ignition times w.

15.5, 15.6, 16.5, 17.5, 19.5, 20.6, 22.8, 23.1, 23.5, 24.5, 26.5, 26.5, 32.7, 33.8, 33.9.

(b) Values of $\log w$: 2.741, 2.747, 2.803, 2.862, 2.970, 3.025, 3.127, 3.140, 3.157, 3.199, 3.277, 3.277, 3.487, 3.520, 3.523.

(c) 14 normalized spacings y: .0063, .1070, .1640, .3912, .2408, .5177, .0752, .1089, .2858, .5724, 0.0, 1.6651, .2676, .0241.

(d) 13 values z: .0014, .0256, .0626, .1510, .2054, .3224, .3394, .3640, .4286, .5579, .5579, .9341, .9946.

$A^2 = 1.878$; $\qquad S = 1 - 0.3394 - 0.661$; $\qquad \bar{Z} = 0.380$.

TABLE 3

Values of K for a large-sample approximation for $\bar{Z}$ (Equation (9)).

Censoring	p	0.0	0.0	0.0	0.0	0.25	0.25	0.25	0.50	0.50	0.75
Fractions	q	0.25	0.50	0.75	1.0	0.50	0.75	1.0	0.75	1.0	1.0
	K	3.467	3.568	3.674	4.010	3.467	3.613	4.002	3.524	3.937	3.854

TABLE 4

Power of 5 test statistics in a 5% test for the extreme-value distribution. The results are the percentage of 5000 samples declared significant by the statistic given. $\bar{Z}$ is a two-tailed statistic. S(1) and S(2) refers to S using one or two tails.

Sample size n = 20

Alternative / Statistic:	A^2	$\bar{Z}$	S(2)	S(1)	A_1^2	R
Exteme-value (null)	5	5	4.6	5	5	5
log W(1,1,1)	80	73	55	66	64	57
log W(1,1,2)	26	26	18	27	18	8
log W(1,1,0.8)	93	87	53	80	82	80
log χ_1^2	8	9	7	2	8	9
log χ_4^2	6	7	5	8	6	3
Double Exponential	47	47	31	42	48	31
Normal	37	39	19	34	32	19
Logistic	29	39	25	29	22	10
Cauchy	84	71	59	52	88	84

Sample size n = 40

Alternative	A^2	$\bar{Z}$	S(2)	S(1)	A_1^2	R
Extreme-value (null)	5	5	4.7	5	6	4
log W(1,1,1)	100	98	90	94	96	95
log W(1,1,2)	61	61	41	55	40	17
log W(1,1,0.8)	100	97	86	91	95	95
log χ_1^2	11	12	9	2	11	10
log χ_4^2	11	12	8	14	8	3
Double Exponential	77	76	61	70	78	46
Normal	68	70	45	59	59	30
Logistic	63	65	51	63	48	18
Cauchy	98	82	73	66	100	98

$W(\alpha,\delta,\gamma)$ refers to the distribution (1). The last four distributions have location parameter 0 and scale parameter 1.

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20. ABSTRACT

In this article are discussed tests for the extreme-value distribution, or equivalently for the two-parameter Weibull distribution, when parameters are unknown and the sample may be censored. The three tests investigated are based on the median, the mean, and the Anderson-Darling A^2 statistic calculated from a set z_i of values derived from the spacings of the sample. The median and the mean have previously been discussed by Mann, Scheuer and Fertig (1973) and by Tiku and Singh (1981). Asymptotic distributions and points are given for the test statistics, based on recently developed theory, and power studies are conducted to compare them with each other and with two other statistics suitable for the test. Of the normalized spacings tests, A^2 is recommended overall; the mean also gives good power in many situations, but can be non-consistent.

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